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# Final Exam: MTH 111, Spring 2025

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54

$$\text{Points} = \frac{\cancel{59}\ 54}{\cancel{59}\ 54}$$

**QUESTION 1. (5 points)** Given two lines $L_1: x = 3t+2,\ y = 2t+4,\ z = 10t+4\ (t \in R)$ and $L_2: x = 6w-7,\ y = 6w-6,\ z = -3w+20\ (w \in R)$. Given that $L_1$ intersects $L_2$ in a point $Q$. Find $Q$.

$L_1: \begin{cases} x = 3t+2 \\ y = 2t+4 \\ z = 10t+4 \end{cases}$ $L_2: \begin{cases} x = 6w-7 \\ y = 6w-6 \\ z = -3w+20 \end{cases}$ *forming equations: $3t+2 = 6w-7$

$\Rightarrow 3t - 6w = -9$

$2t+4 = 6w-6$

$2t - 6w = -10$

$t = 1 \quad w = 2$

*substituting into z:

$z = 10(1)+4 = 14 = z = -3(2)+20 = 14$

*finding the point: the lines do intersect

$x = 3(1)+2 = 5$

$y = 2(1)+4 = 6$

$\boxed{(5, 6, 14)}$

**QUESTION 2. (9 points)**

Given that $q_1 = (0, 4, 8)$, $q_2 = (2, 1, -6)$, and $q_3 = (-6, 8, 2)$ are not co-linear.

(a) Find a vector $F$ that is perpendicular to both vectors $\overrightarrow{q_1q_2}$ and $\overrightarrow{q_1q_3}$.

$\overrightarrow{q_1q_2}: \langle 2, -3, -14 \rangle$ $\quad \overrightarrow{q_1q_3}: \langle -6, 4, -6 \rangle$

$$\vec{F} = \overrightarrow{q_1q_2} \times \overrightarrow{q_1q_3} = \begin{vmatrix} i & j & k \\ 2 & -3 & -14 \\ -6 & 4 & -6 \end{vmatrix} = \left\langle \begin{vmatrix} -3 & -14 \\ 4 & -6 \end{vmatrix}, -\begin{vmatrix} 2 & -14 \\ -6 & -6 \end{vmatrix}, \begin{vmatrix} 2 & -3 \\ -6 & 4 \end{vmatrix} \right\rangle$$

$\vec{F} = \langle (-6 \times -3) - (-14 \times 4), -((2 \times -6) - (-14 \times -6)), (2 \times 4) - (-3 \times -6) \rangle$

$\boxed{\vec{F} = \langle 74, 96, -10 \rangle}$

(b) Find the equation of the plane, say $P$, that passes through $q_1, q_2, q_3$.

$\vec{F} = \langle 74, 96, -10 \rangle \rightarrow N$

$74x + 96y - 10z = C$

$74(0) + 96(4) - 10(8) = C \quad C = 304$

$\boxed{74x + 96y - 10z = 304}$

(c) Find a vector $F$ such that $|F| = 57$ that is perpendicular to both vectors $\overrightarrow{q_1q_2}$ and $\overrightarrow{q_1q_3}$.

$\Rightarrow \dfrac{|F|}{|q_1q_2 \times q_1q_3|} \times F \quad \Rightarrow \dfrac{57}{\sqrt{74^2 + 96^2 + (-10)^2}} \times \langle 74, 96, -10 \rangle$

$\Rightarrow \boxed{\dfrac{57}{86\sqrt{2}} \langle 74, 96, -10 \rangle}$

**QUESTION 4. (5 points)** Find the equation of the tangent line to the curve $f(x) = e^{x-2} + 60x + 3$ at $x = 2$.

* the point :

$f(2) = e^{2-2} + 60(2) + 3 = 124$ $\boxed{(2, 124)}$

* the slope:

$f'(x) = (1)\, e^{x-2} + 60$

$f'(2) = e^{2-2} + 60 = 61$ $\boxed{m = 61}$

* b: $y = mx + b$

$124 = 61(2) + b$ $\boxed{b = 2}$

* the equation : $\boxed{y = 61x + 2}$

**QUESTION 5. (9 points)** Find $f'(x)$ but do not simplify

(i) $f(x) = (e^{3x})(3x+7)^2$.

$f'(x) = (3e^{3x})(3x+7)^2 + (e^{3x})(3)(2)(3x+7)$

(ii) $f(x) = ln[(7x+2)^4] + 43x + 1$ $\quad f(x) = 4\ln(7x+2) + 43x + 1$

$f'(x) = \dfrac{4(7)}{7x+2} + 43$

(iii) Let $k(x) = f(2x-10)$ such that $f'(0) = 25$. Find $k'(5)$.

$k'(x) = (2)\, f'(2x-10)$

$k'(5) = 2 f'(2(5)-10) \Rightarrow k'(5) = 2 f'(0)$

$\boxed{k'(5) = 2 \times 25 = 50}$

~~**QUESTION 6. (5 points)** Find the equation of the tangent line to the curve $f(x) = e^{x-2} + 60x + 3$ at $x = 2$.~~

**QUESTION 7. (12 points)** Let $f(x) = 2x^3 - 12x^2 + 1$.

(i) For what values of $x$ does $f(x)$ increase?

$f'(x) = 6x^2 - 24x = 0$

$x_1 = 4 \quad x_2 = 0$

$+ \quad - \quad +$

$0 \quad 4$ sign of $f'(x)$

$f(x)$ increases between $\boxed{(-\infty, 0) \cup (4, +\infty)}$

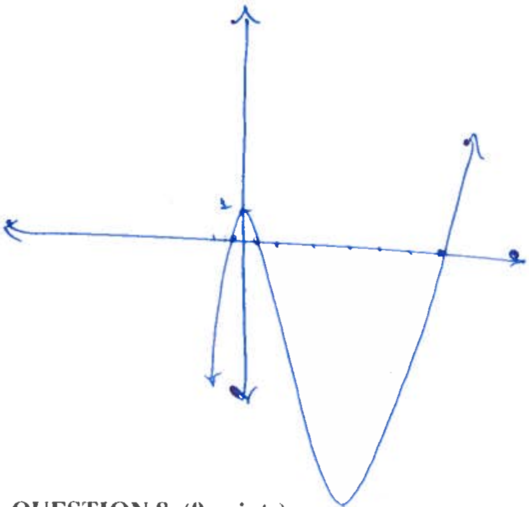
(ii) For what values of $x$ does $f(x)$ decrease?

$f(x)$ decreases between $\boxed{(0, 4)}$

(iii) Find all $x$ values where maximum and minimum occur.

* $f(x)$ has a local max at $\boxed{x = 0}$ $f(0) = 1$ $\boxed{(0, 1)}$
* $f(x)$ has a local min at $\boxed{x = 4}$ $f(4) = -63$ $\boxed{(4, -63)}$

(iv) Sketch the graph of $f(x)$.




**QUESTION 8. (9 points)**

(1) $\int \frac{x+3}{x^7}\, dx$

$\Rightarrow \int \frac{x}{x^7} + \frac{3}{x^7}\, dx \Rightarrow \int x^{-6} + 3x^{-7}\, dx$

$\Rightarrow \boxed{-\frac{x^{-5}}{5} - \frac{3x^{-6}}{6} + C}$

(2) $\int 3x^2 + 8x^3 + 2x - 7\,dx$

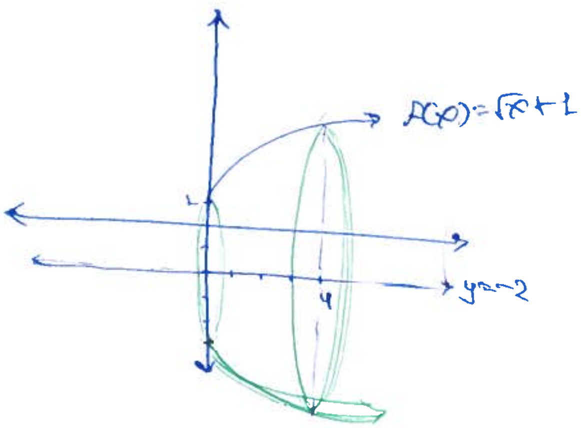
$\Rightarrow \frac{3x^3}{3} + \frac{8x^4}{4} + \frac{2x^2}{2} - 7x + c$

$\Rightarrow \boxed{x^3 + 2x^4 + x^2 - 7x + c}$

(3) $\int (\frac{1}{x^3} + \sqrt{x^3} + 10)\,dx$ $\Rightarrow \int x^{-3} + x^{\frac{3}{2}} + 10 \cdot dx$

$\Rightarrow \boxed{\frac{-x^{-2}}{2} + \frac{2x^{\frac{5}{2}}}{5} + 10x + c}$

**QUESTION 9. (5 points)** Find the volume of the object after rotating $f(x) = \sqrt{x} + 1$ about $y = -2$. $0 \le x \le 4$




$V = \int_0^4 \pi r^2 \cdot dx$

$V = \pi \int_0^4 (\sqrt{x} + 1 + 2)^2 \cdot dx$

$V = \pi \int_0^4 (\sqrt{x} + 3)^2 \cdot dx \Rightarrow V = \pi \int_0^4 (x + 6\sqrt{x} + 9) \cdot dx$

$V = \pi \left(\frac{x^2}{2} + (6)\frac{2x^{\frac{3}{2}}}{3} + 9x\right)\Big|_0^4 = (76 - 0)\pi$

$\boxed{V = 76\pi = 238.761 \text{ units}^3}$

Faculty information